Abstract—This paper deals with the focusing of low resolution ScanSAR data, for both detected amplitude images and interferometric applications. The SAR reference is exploited to achieve ScanSAR focusing in conventional techniques. Such techniques provide quite effective compensation of the azimuth antenna pattern (AAP) (e.g., no scalloping) when the azimuth time-bandwidth product (TBP$_{az}$) of the ScanSAR echo is large, but fail to do so as the burst shortens, being reduced to an ineffective weighting of the output. The result is an azimuth varying distortion of the focused impulse responses, a distortion that is partly compensated for in the multilook average (not available for interferometric applications) at the price of a reduction in the processed Doppler bandwidth.

This paper proposes quite a different approach. A set of short kernels, each suitable for “focusing” at a specific azimuth bin, has been optimized to reconstruct source reflectivity in the minimum mean square error (MMSE) sense. That pseudoinversion converges to the “conventional” focusing when the burst extent is large and for short bursts, edge effects are accounted for. These azimuth-varying kernels can be suitably tuned to meet constraints in the resolution/sidelobes trade-off and have proved capable of providing fairly undistorted output and fine resolution. They better exploit the available Doppler bandwidth, maximizing the number of looks and the interferometric quality. A decomposition is suggested that implements the inverse operator as a fast preprocessing to be followed by a conventional ScanSAR processor.

I. INTRODUCTION

S

canSAR is a particular synthetic aperture radar (SAR) that achieves very wide swath coverage by periodically switching the antenna pointing in several range “subswaths” [1], [2]. This unique feature makes ScanSAR a very useful mode, causing it to be included in most recent and forthcoming spaceborne SARs (SRTM, RADARSAT, ASAR). The principle of ScanSAR is shown in Fig. 1(a), which shows the geometry of a five subswath ScanSAR like that of the future ENVISAT ASAR sensor.

The ScanSAR sensor acquires short “bursts” of time extent $T_D$ (“dwell time”), by cyclically scanning all the subswaths. The continuous coverage of the scene requires that at least one burst is acquired for the time extent of the antenna beam width $T_F$ (“footprint time”). Each target is imaged in a burst within a small range of antenna apertures, as the Fig. 1 shows. Thus, the received echo is weighted by the antenna beam in a way that depends on the scatterer position. The resulting azimuthvariant amplitude, responsible for scalloping [2], [3] is shown in Fig. 1(b). Such scalloping is perhaps the major ScanSAR drawback, but this can be compensated by averaging several “looks,” e.g., by imaging each target several times from different view angles. The duration of each burst is thus limited to an interval $T_D < T_F/(N_a N_p)$ ( $N_a$ being the number of subswaths and $N_p$ the number of bursts in a footprint). This constraint implies an additional limit to azimuth resolution and is the price paid for the increased ScanSAR coverage and the reduction in scalloping. For example, in wide swath mode (WSM) the ASAR sensor acquires $N_a = 3$ looks for each of the $N_p = 5$ subswaths, resulting in an azimuth resolution of 150 m (30 times worse than the SAR mode). The factor 30 is the result of $N_a N_p$ plus a necessary margin that accounts for data loss in the switching and a proper gap. The same sensor also operates in very low resolution global monitoring mode (GMM), where the burst is reduced to a few samples ($7 \div 10$) to keep data rate low.

The focusing of ScanSAR data (eventually phase preserving) is usually performed by exploiting the SAR matched-phase reference, similar to conventional SAR focusing. Several efficient techniques that have been developed provide different implementations of the same transfer function [4]. Each technique is based on a specific SAR focusing scheme: “SPECAN” (the most efficient), “range-Doppler,” “chirp-scaling,” “$\omega - k$,” “chirp-Z transform,” etc. [5]–[8].

The term “focusing,” however, may be misleading when used within the context of ScanSAR data processing. If the burst is long enough to achieve a large TBP$_{az}$, the response to a point scatterer after correlation is, in effect, shorter than the input burst. However, if TBP$_{az} \sim 1$, the output response may easily be wider than the input burst. Nonetheless, correlation with a chirp replica is fundamental to separate the echoes from different targets and to placing each echo at its correct azimuth position. This procedure is better defined as “inversion,” in the sense that an estimate of source reflectivity (say over the extent of an antenna footprint) can be made from each ScanSAR burst.

In conventional SAR, the echo has a quadratic phase and is weighted by the AAP. In ScanSAR, the echo is the same, except for the “chopping” effect due to the scanning mechanism. This makes the system a time-varying one, as the AAP weights the echo of each target in such a way that it depends on target location [see Fig. 1(b)]. ScanSAR inversion should therefore entail a time-varying processing, where a different kernel would be exploited for each different output azimuth bin. The impulse responses scattered by all the targets close to an azimuth bin should be properly accounted for by each kernel. As these impulse responses have different shapes, the design of the kernel becomes a nontrivial task.

The large TBP$_{az}$ ScanSAR case is much simpler since the spectrum of each target in effect is simply a windowing of the “long” SAR impulse response function (IRF). Accordingly, the proper set of focusing kernels is computed by windowing the inverse SAR reference, the SAR PCF [9]. This result is a conse-
The approximated ScanSAR impulse response we will refer to is shown in Fig. 1. In particular, let $t$ be the azimuth (slow time) coordinate in the “signal domain” and $\xi, \eta_D$, respectively, the azimuth time and range location (closest approach, hence zero Doppler) of a scatterer (in the “data domain”). Let us assume that the sensor moves along a straight path, parallel to the ground, with velocity $v$, and fixed targets. The approximated ScanSAR geometry we will refer to is drawn in Table I.

The ScanSAR IRF, i.e., the signal backscattered by a point target at $(r_D, \xi)$, can be approximated, in continuous time, as follows [6] (a zero Doppler centroid has been assumed):

\[ h_s(t; \xi) = a_\beta(t - \xi) \text{rect} \left( \frac{t}{T_D} \right) \exp \left( j \pi f_R (t - \xi)^2 \right). \quad (1) \]

In the notation $(t; \xi)$, we have emphasized the nonstationarity of ScanSAR acquisition, as targets at different azimuths result in different signals $h_s(t)$ (and the difference is not simply a shift). The three terms on the right side of (1) have the following meanings.

- $a_\beta(t)$ is the AAP and depends on the angle $\beta = \frac{v \xi / \sqrt{v^2}}{\sqrt{v^2}}$ between the zero Doppler axis and the sensor-target direction.
- Rect $\left( \frac{t}{T_D} \right)$ accounts for the burst mode acquisition of ScanSAR.
- The complex exponential is the Doppler phase history of the target, proportional to the Doppler rate $f_R$.

Note that (1) refers to a single “burst.” The coordinate system has been chosen so that slow time $t = 0$ refers to the center of the burst, and the target at $\xi = 0$ is the one at the center of the antenna beamwidth at burst center.

Here, the very low range and azimuth resolutions allow for monodimensional assumption (in the ASAR ScanSAR case, the maximum range migration is $<1/8$ of a resolution cell).

The effect of rectangular burst windowing, that is specific to ScanSAR, is twofold. First, when combined with the AAP, $a_\beta(t)$, the first factor in (1), it causes an azimuth varying amplitude of the IRF, known as scalloping, already shown in Fig. 1(b). Second, when combined with the Doppler phase history, the third factor in (1), it causes different portions of the complex reflectivity spectrum to be imaged at different azimuth positions [6]. This implies that the same scatterer contributes to
different bursts with different amplitudes (scalloping) and different azimuth frequency bands. For this reason, the ScanSAR data are focused burst by burst, and then results are either detected or averaged to obtain multilook amplitudes, or coherently averaged, at the interferogram level, for interferometric applications [10].

III. CONVENTIONAL ScanSAR FOCUSING FOR LOW TBP_{ax}

Techniques known in the literature [5]–[8] implement ScanSAR focusing by extending the conventional SAR focusing technique, e.g., by means of the matched phase SAR reference

\[ h_{is}(t) = \exp(j \pi f_R t^2). \]  

(2)

The contribution of a target at azimuth \( \xi \) to the focused image is obtained by correlation

\[ \hat{h}(t; \xi) = h_{is}(t; \xi) * h_{is}^*(-t) \quad - \frac{T_F}{2} \leq t \leq + \frac{T_F}{2} \]  

(3)

\[ = \int_{-\infty}^{+\infty} u(\tau; \xi) \cdot \exp(j \pi f_R (\tau - \xi)^2) \exp(-j \pi f_R (\tau - t)^2) \, d\tau \]
\[ = \exp(-j \pi f_R (t^2 - \xi^2)) \int_{-\infty}^{+\infty} u(\tau; \xi) \cdot \exp(j 2 \pi f_R (t - \xi) \tau) \, d\tau \]  

(4)

where \( \ast \) denotes convolution, and \( u(\tau; \xi) = a_{\beta}(\tau - \xi) \) \( \text{rect} (\tau/T_D) \) is the AAP shifted and windowed by the burst envelope. Even though the same long replica \( h_{is}(t) \) is used to “focus” all the targets, different portions of this replica (i.e., windowed by the burst envelope: \( \text{rect}(\tau/T_D) \)) make a significant contribution to the output from targets at different azimuth positions.

Let us assume the simple case when AAP is constant within the burst duration \( T_D \). This case fits fairly well the ASAR GMM, where the burst lasts for just a few echoes. If \( u(\tau; \xi) \simeq a_{\beta}(\tau - \xi) \) \( \text{rect} (\tau/T_F) \) (assuming window \( u_b \) for burst data), the integral in (4) becomes a FT. Thus, see (5), shown at the bottom of the page.

Whereas at the edges of the synthetic aperture (for \( |t| \simeq T_F/2 \)), oscillatory transitions of extent \( T_D \) result. The focusing of one ScanSAR burst is represented in Fig. 2. That burst is the superposition of the returns of the three scatterers drawn in Fig. 1(b).

The burst window FT, \( W_b \), plays an important role as it fixes the “shape” of the FIRF amplitude [2] in (5). Hence, it can be designed to fulfill constraints on resolution, sidelobes etc. Note that the FIRF \( \hat{h}(t; \xi) \) would be invariant (depending only on \( t - \xi \)), except for an azimuth varying frequency shift and an azimuth varying scale factor [corresponding, respectively, to the second and the third factors in the right member of (5)]. The azimuth varying scale factor \( a_{\beta}(\tau - \xi) \) (due to the AAP) results in an amplitude variation that is conventionally defined “scalloping.”

This conventional focusing scheme can be easily implemented, in the actual sampled case, by substituting the discrete variables \( t_k, \xi_l \) for the continuous versions \( t, \xi \). If a scene of \( M \) azimuth bins (in the whole footprint) is to be reconstructed out of a burst of \( N \) echoes, e.g., we sample output at a rate \( \Delta T = T_F/M \); we get the following continuous to discrete time mapping:

\[ t_k = k/P \text{RF} - T_D/2 \quad \text{for} \quad k = 0 \cdots N - 1 \]
\[ \xi_l = l \cdot \Delta T - T_F/2 \quad \text{for} \quad l = 0 \cdots M - 1. \]  

(6)

Azimuth focusing is then performed by a matrix multiplication

\[ \hat{X} = H_{is}^t B_{aw} \]  

(7)

between the windowed burst \( B_{aw} \) (a column vector) and the matrix of kernels \( H_{is}^t \), to be iterated for each burst and each range bin. We use here bold capitals to represent vectors or matrixes

\[ \hat{h}(t; \xi) \simeq \begin{cases} \exp(j \pi f_R (t - \xi)^2) \exp(j 2 \pi f_R (t - \xi) \xi) a_{\beta}(\tau - \xi) W_b(-f_R (t - \xi)), & \text{for} \ |t| < \frac{T_F - T_D}{2} \\ 0, & \text{for} \ |t| > \frac{T_F + T_D}{2} \end{cases} \]  

(5)
and "*" for conjugate transposition. The matrix sizes and other useful scalar parameters are defined in Table II. Each row of the matrix $\mathbf{H}_s$ keeps the reference for focusing at a specific azimuth bin: the generic element $h_{is}(\xi, t)$ of that matrix is the conjugate of $h_{is}(t)$ in (3) evaluated at $t = t_k - \xi$. This ScanSAR focusing scheme is represented in Fig. 3. Note that when the burst is very short this time domain focusing becomes quite efficient, like any other FFT based technique. For example, in ASAR GMM, the implementation of (7) requires seven to ten multiplications per output sample (comparable with the cost of a single FFT).

The result expected from this discrete time domain focusing is close to sampling the continuous time FIRF in (5), the sole difference being the folding of the burst window $F(t)$, with period $PRF$. However, the sampled nature both of the input signal and of the processing introduces “aliasing,” and the contribution of targets located close to one edge of the footprint appears as a ghost at the other edge. The only remedy for this aliasing is to limit the extent of a focused burst to roughly

$$\Delta T_{az} \leq PRF \frac{1}{PRF - \Delta T_{az}}$$

being the azimuth lobe width. For example, in the ASAR GMM mode, this implies a limit to processed Doppler bandwidth equal to \( \sim 85\% \) of the PRF.

### A. AAP Compensation

ScanSAR focusing by the constant amplitude SAR matched reference, assumed until now, results in an azimuth-variant amplitude, or scalloping, due to the AAP [2], [3], [11].

In classical SAR, the AAP may be compensated to obtain a desired FIRF shape (according on some constraints on PSLR, ISLR, etc.). In ScanSAR, where different targets experience weighting from different segments of AAP, the reflected echoes vary in both amplitude and shape. Hence conventional compensation is not possible.

The classical way to compensate for AAP is to single out from the input data all the contributions coming from a well defined direction and to scale them by the inverse of the AAP in that same direction. The only way to identify these contributions is through their Doppler shift, but good estimation of the Doppler shift requires much better spectral resolution than the signal bandwidth. Therefore, as spectral resolution is inversely proportional to observation time, good antenna compensation in the range-Doppler domain calls for high $TBP_{az}$. Obviously, this holds for both classical SAR and ScanSAR.

As long as $TBP_{az} \gg 1$, it is well known that applying the inverse antenna weighting to the signal azimuth spectrum is equivalent to applying it to the chirp reference. In medium resolution ScanSAR focusing and AAP correction are achieved at one time by means of the SAR PCF reference, that is inversely weighted by the AAP.

The approach does not work in the low $TBP_{az}$ case, even though it is used altogether [7]. This can be shown by modifying (2) to account for the SAR “inverse” or the PCF reference. $h_{is}(t)$ must be weighted by the inverse AAP $a_{\beta}^{-1}(t)$, and this results in a factor $a_{\beta}^{-1}(\tau - t)$ in the integral in (4). Here again, the burst is assumed so short that $a_{\beta}^{-1}(\tau - t) \approx a_{\beta}^{-1}(-t)$ and this term can be moved outside the integral. This is the “bad” point. Instead of deconvolving AAP, we end up in weighting the focused data by the inverse AAP:

$$\left| W_e(t \pm \xi) \right| a_{\beta}^{-1}(\tau - t) \approx a_{\beta}(-\xi) a_{\beta}^{-1}(\tau - t)$$

The first factor in the FIRF amplitude (8), $\left| W_e(-f_R(\cdot)) \right|$, is azimuth invariant (e.g., it depends only on $t \pm \xi$), but, unfor-
Fig. 4. (a) FIRF (acquired by SAR constant amplitude reference) of three targets at different azimuth $\tau / \xi$ ($\tau / \xi = 0$ identifies the beam center). The different amplitude is due to scalloping. (b) When scalloping is compensated by the “inverse” reference, e.g., PCF, an azimuth varying distortion arises. It corresponds to multiplying output (a) by the inverse of the AAP.

Interestingly, not the others. Only at the nominal target position for $t = \xi$ do we get the proper compensation $\alpha_3(\xi)\alpha_3^{-1}(t) = 1$, i.e., no scalloping. However, the term $\alpha_3^{-1}(t)$ distorts the FIRF shape asymmetrically and shifts its peak, introducing an “apparent” scalloping.

An example of FIRF distortion expected in the GMM mode is provided in Fig. 4. No burst window was superimposed on the rectangular scan pattern to get a “sinc” shaped FIRF, according to (5), and thus obtaining the finest azimuth resolution $\delta_{az} = (f_H T_D)^{-1} \approx m$ and PSLR of $\sim -13$ dB. The inverse antenna weight $\alpha_3^{-1}(t)$ introduces marked asymmetries in target response on the right (located at $\sim 1/6$ of the footprint) and on the left, where an apparent scalloping of $12\%$ ($\sim 1.6$ dB) can be measured. Note also the increase of up to 5 dB in the FIRF sidelobes (for the scatterer on the right), and the effect of aliasing evident in the wrap-around of the FIRF of the leftmost scatterer.

B. Optimizing Conventional ScanSAR Focusing

So far, we notice that, in processing a single burst, scalloping can be compensated (in as far as the peak amplitude of the FIRF is concerned) only by weighting the SAR replica by the inverse AAP, at the price of an azimuth-varying CNR and marked FIRF distortion. The sole mode to counteract scalloping and FIRF distortion is in a multilook environment: averaging the intensity images cancels out asymmetric distortions resulting in an almost invariant FIRF with constant amplitude and symmetric shape.

The maximization of overall ScanSAR quality was achieved in [12], the problem of getting constant amplitude (no scalloping) by combining multiple looks being solved by tailoring the azimuth varying look weight, making a tradeoff between equivalent number of looks ($ENL$) or radiometric resolution, and noise equivalent $\sigma_0$, or CNR. Thus, the only possibility that scalloping occurs is due to the uncertainty of Doppler centroid. However, this can be kept under control by using techniques that leave a residual radiometric error of a fraction of dB [3]. The approach proposed in [12] is quite effective as long as the azimuth weighting window is so smooth to be assumed constant within some resolution cells. The rationale is that the output FIRF is short enough not to be distorted by the azimuth varying weight. In low TBP ScanSAR, the porting of this solution is not so simple. In GMM mode, for example, the FIRF spreads over $\sim$one-fifth of the whole footprint (if limited to the optimistic assumption of $-3$ dB fall-off). Here, the design of the “azimuth weighting window” becomes quite complicated, since that window affects FIRF distortion (see (8)), and hence resolution, sidelobes, etc.

In the GMM mode, the CNR is quite good as small range bandwidths are used$^1$, the “optimal” solution would be close to the one that maximizes the ENL, this being achieved by correcting the AAP over the whole Doppler bandwidth while processing each burst. However, as we have shown, this is not possible since the FIRF distortion would increase the sidelobes to inadmissible levels. Finally, a compromise is reached, where the processed Doppler bandwidth is traded for lower FIRF distortion. An example of such solutions is given in Fig. 5. The figure plots the (distorted) FIRF obtained by maximizing ENL. Compared with these, the more regular FIRF resulting by the use of a tapered reference (that is equivalent to taper the focused burst, as discussed in Section III-A) represents a more reasonable trade-off.

IV. ScanSAR PSEUDO-INVERSION

So far, we have shown that “conventional” ScanSAR focusing is unable to provide a space invariant, undistorted FIRF. Moreover, any attempt to maximize quality by optimizing the look weighting window (as discussed in [12]), would result in further FIRF distortion. In the previous section, we have shown how a compromise can always be found, but at the price of a reduced footprint (hence a loss in radiometric resolution).

A better way is to approach low time bandwidth ScanSAR processing as an azimuth-varying inversion. In this framework, the “look weighting window” discussed in Section III-B

$^1$This is achieved by reducing the transmitted chirp FM rate, without changing its duration.
provides the simplest solution, a zero-order time-varying processing, yet this approach is shown to be ineffective in low TBP.

The focusing of SAR data can be simply formulated as a matrix inversion problem by modeling the SAR acquisition in the discrete time\(^2\) as \(B = H_sX\). The source reflectivity \(X\) is to be retrieved by the collected data \(B\) (that is no longer a burst) by inverting the (known) acquisition matrix \(H_s\). This is quite possible as long as the data vector size and the matrix size agree. With ScanSAR, data vectors are far shorter than it would be required for a good inversion. Conventional algorithms artificially make vector \(B\) longer, through zero padding. A different approach is to pursue pseudo-inversion: as no true solution exists [13], a pseudo-solution that gives the least squared deviation from data is searched, instead. In this procedure, it is customary to allow for imprecise or noisy knowledge of the data vector.

Let us assume the following model for the ScanSAR raw (or range compressed) data burst: \(B = H_sX + N\), where noise \(N\) has been added to the burst data, \(B\) (of size \([N, 1]\)). According to the notation in Section III, \(X [M, 1]\) represents the scene reflectivity and \(H_s[N,M]\) the ScanSAR IFR matrix (each column containing the contribution of a scatterer to the burst) whose elements \(h_s(k,l)\) are derived from (1) evaluated for \(t_k - Qk\).\(^{[48]}\) and \(Q\) defined in (6).

It is shown in Appendix A that, even in the no noise case, a unique inverse solution to ScanSAR acquisition exists only for \(T_D \rightarrow \infty\), or reasonably, for large burst extents. As the burst reduces, the only possibility is to look for approximate solutions. The classical SAR focusing is, in effect, a MMSE solution in itself [9]. Additional “regularization” criteria can be found in the literature on the solution of integral equations. Therefore, we deem it appropriate to formulate the problem of ScanSAR inversion in discrete time as the retrieval of the reflectivity \(X\) at a distance closest to the unknown source \(\tilde{X} \simeq X\), in some norm.\(^{[48]}\)

Despite the simple formulation, the solution to the problem in a probabilistic framework is quite complicated (from both a formal and a computational point of view), involving a general time variant estimator. However, in the case that both \(X\) and \(N\) come from gaussian stationary processes, the minimum variance unbiased estimator is the solution of the linear MMSE problem [15]

\[
\min_{\tilde{X}} E[(X - \tilde{X})(X - \tilde{X})^*] = H_{\text{inv}}^*H_{\text{inv}}B.
\]

\(H_{\text{inv}}\) is provided by Wiener pseudo-inversion [15]

\[
H_{\text{inv}} = (H_sR_xH_s^* + R_n)^{-1}H_sR_x\]

\(R_x\) being the covariance matrix of the scene reflectivity, \(X\), and \(R_n\) the covariance matrix of noise \(N\). MMSE ScanSAR inversion is thus achieved by the following linear time variant (LTV) filtering:

\[
\tilde{X} = \left((H_sR_xH_s^* + R_n)^{-1}H_sR_x\right)^*B = R_xH_s^*\left((H_sR_xH_s^* + R_n)^{-1}\right)B.
\]

It is interesting to note that, when \(R_n = 0\) and \(R_x = I\), the proposed MMSE inversion becomes the least square (LS) pseudo-inversion of the ScanSAR acquisition. The reader may find further details in the Appendix B.

A. Implementation

The MMSE solution is obviously not a unique solution. It is only the best given the problem constraints in terms of mean square error (MSE). This does not guarantee it to be the “optimal” one for any ScanSAR scene. Indeed, its most attractive feature, from our point of view, lies in the providing of “inversion” in a linear filtering framework where the set of time-varying references can be efficiently computed by matrix inversion. Even so, one good point seems the fact that MMSE is optimal (in probabilistic sense) for retrieving an homogenous,
normal distributed target in Gaussian noise. In fact, this is the case of interest for many potential users of very low resolution SAR imagery, as in the study of arctic ice. Furthermore, homogenous speckle is the kind of target mostly implied in SAR interferometry literature (see: [16], [17], and many others). MMSE ensures orthogonality between the error and the reconstructed reflectivity, thus maximizing coherence i.e., the normalized cross-correlation coefficient, a well assessed measure of interferometric quality. Moreover, the convergence of MMSE to conventional ScanSAR focusing for TBP \( \approx 1 \) (see details in the Appendix B) validates this as an interesting solution.

The finest resolution can be achieved when \( \mathbf{R}_x \) is diagonal as this implies impulsive autocorrelation, i.e., the largest bandwidth. The noise covariance matrix is always diagonal. For a white noise of power \( P_n \), we get \( \mathbf{R}_n = P_n / \lambda L \), where \( P_n \) can be dimensioned according to the actual raw data CNR \( \text{(12)} \). It is shown in Appendix B that assuming larger CNR does little to change the solution.

Matrix \( \mathbf{H}_x \), which describes the direct problem, is critical for the Doppler centroid. In practice, an appropriate Doppler centroid retrieval technique is needed. The algorithms currently available are so accurate (20 Hz mismatch, corresponding to 0.25 dB in radiometric calibration error [3]), that no detectable artifact appears.

Basic MMSE inversion (11) leads no control on FIRF side-lobes, and this could be a serious problem. Like the actual implementation of the PCF [9], some tuning is mandatory to meet constraints on pulse response, in terms of peak side-lobe ratio (PSLR), integrated side-lobe ratio (ISLR), etc. We can, however, take advantage of the result in (5), where it is shown that the FIRF shape can be tuned by properly designing the burst window. This result still holds for the MMSE inversion, provided that the low TBP \( \approx 1 \) assumption holds (see details in the Appendix B). Obviously, the window has to be applied during focusing, and not while computing the inverse operator, otherwise the inversion procedure would try to get rid of it.

The values on the diagonal \( \mathbf{R}_x \) fix the desired square amplitude of the reconstructed reflectivity along the azimuth. The tuning of these values compares with the azimuth look weighting window approach, discussed in Section III-B. However, the matrix \( \mathbf{R}_x \) provides the proper separation of the data domain (e.g., the peak amplitude \( |x(t, \xi)|_{T \geq 0} \), from the signal domain (e.g., the FIRF shape for \( t \neq \xi \)). If AAP correction is desired over the whole footprint, as is probably the case with the GMM mode, all the elements on the diagonal should be assigned the same value, causing \( \mathbf{R}_x \) to be an identity matrix.

Further details on the capabilities of MMSE to remove the FIRF distortions are given in the Appendix B together with an efficient implementation of the technique as preprocessing to conventional ScanSAR focusing.

An example of the achievable FIRF that results from MMSE is in Fig. 6(a), to be compared with Fig. 5 (conventional focusing). Aliased contributions are indicated in the plot: they can be easily removed by windowing the output—as discussed in Section III. The advantage in terms of resolution in a multilook environment can be guessed from Fig. 6(b), that compares the FIRF of scatterers at different azimuth obtained through MMSE and conventional focusing. Note that the 3 dB lobe width is approximately the same, however, in the second case it is asymmetrically distorted. The multilook combination is effective in reducing asymmetries in the conventional case, but at the price of a mainlobe broadening, i.e., a resolution loss.

A quantitative comparison of the performance achievable through ScanSAR inversion techniques in a multilook environment (GMM case) is shown in Fig. 7. Here we have considered three focusing schemes. First, MMSE \( 7 \rightarrow 2 \) refers to MMSE inversion where the output has been limited to \( \approx 7/9 \) of the footprint (e.g., seven interburst intervals over nine) to avoid aliasing. Second and third are SAR PCF focusing, weighted by a window flat for, respectively, five and seven interbursts,
and tapered to zero thereafter. Clearly the first and third cases provide the same ENL, since they exploit the same Doppler Bandwidth, whereas the second case shows lower performance. These results are reported in terms of PSLR and resolution for different burst windows: we used a generalized Hamming window, with parameter $\alpha$ ranging from 0.7 to 1 (1 means boxcar windowing). Note that, at the same resolution and ENL, MMSE inversion achieves an improvement of 3 dB in PSLR. Alternatively the gain in resolution at the same sidelobes levels is in the order of 5% (a 4.5% resolution loss and some PSLR decrease is reported in [7], in the case of SIR-C ScanSAR data processed by exploiting the “usual” SAR PCF reference).

An example of a simulated ScanSAR data-set from ERS-1, focused in WSM and GMM mode by the MMSE algorithm is shown in Fig. 8. The measures of PSLR, ENL, resolution made on the data-set and compared with the standard focusing confirmed the expected improvements.

V. OPTIMIZING INTERFEROMETRIC QUALITY

Currently it is not possible to discuss a new SAR “focusing” algorithm without assessing its usefulness in interferometric applications. In maximizing the interferometric quality, one has to take into account, besides the usual thermal, quantization, and
ambiguity noises, other noise sources specific to that kind of application. These noises derive from variations in the imaged reflectivities of the two acquisitions. They are mainly due to temporal changes in the repeat-pass interval, and also to the acquisition geometry (baseline, crossed orbits), volume scattering, etc. [18], [19], [17]. These scene noise sources add up to the target reflectivity, quite differently from the other system noises that add up to acquired data, as Fig. 9(a) shows.

The contribution of these noises to the focused image is plotted in Fig. 9(b), with the assumption of complete AAP deconvolution (perfect descalloping), and a homogeneous distributed scatterer. In the plots, system noise giving a CNR of 10 dB in the best case (at the beam center) has been assumed, whereas two different values of scene noise are reported, corresponding to scenes of “good” coherence (γ = 0.7), and very good (γ = 0.9). In the case drawn in the figure, we have assumed an ideal focusing, e.g., we have ignored the FIRF distortion and the processor induced aliasing discussed in Section III. Clearly, the signal to scene noise ratio, SNR is invariant with azimuth, but the signal to system noise ratio (SNR$_{{sys}}$) changes. Like the case discussed in Section III-B, SNR$_{{sys}}$ that results in the multilook averaged interferogram can be tuned by fixing the extent of the output footprint, e.g., look weighting window. However, there is no way to counteract for scene noise, except by averaging a higher number of looks, e.g., getting full AAP compensation. According to the result plotted in Fig. 9(b), it appears evident that the AAP compensation is indeed the best strategy up to ~70% of the footprint, where scene noise dominates, also in the case of very good (non decorrelated) scenes. In this sense, the proposed MMSE focusing complies with the maximization of interferometric quality.

A. Phase Preserving Processing

An important factor of the final interferometric quality is processor phase distortion. In low TBP, ScanSAR interferometry, a decorrelation source is introduced by a different FIRF shape in the two coregistered focused images. This may be the case when the two AAP are shifted or when one of the two images is synthesized from a SAR focused image [20]. This decorrelation can be expressed in terms of the normalized correlation coefficient between the two FIRFs and can then be converted to phase noise according to [18], [19], [17]. It can be measured by adapting the CEOS phase-preserving test described in [21], [22]. For example, the processor induced decorrelation was computed for both ASAR GMM and WSM cases. The result of the CEOS test$^3$ in the case of the focusing performed by the MMSE technique and the conventional one (with a reference window that is flat and has cosine roll-offs at the edges) is shown in Fig. 10. In the GMM, the decorrelation is stronger, since the FIRF width is close to the footprint (hence aliasing dominates). In this case the MMSE solution gives a decorrelation comparable to that achieved by a cosine windowed reference, however MMSE performs better since it explores a larger bandwidth. For the WSM case, where the resolution is rather fine, the MMSE technique gives better results than conventional techniques, due to the stationary FIRF, as Fig. 10(b) shows. If compared with the results achieved by a mostly flat reference (flat for 75% of the footprint and windowed by a cosine transition for the remaining 25%), a phase noise of 8° (SNR = 17 dB) is measured at 1/4 of the footprint instead of 14° (SNR = 24 dB). This compares favorably with the 5.5° phase noise limit imposed by CEOS test for full resolution SAR interferometry.

VI. CONCLUSIONS

The problem of ScanSAR inversion, e.g., retrieving an estimate of source reflectivity from the backscattered field, has been approached in the case of low resolution. When TBP$^a$ becomes small, conventional ScanSAR focusing ceases to be an inversion. The output FIRF shape becomes affected by an azimuth

$^3$The CEOS test basically checks the space-invariance of the processor. In the simulations, only an azimuth shift was assumed.
varying distortion. This distortion can be compensated in a multilook environment, but not for phase preserving focusing, by optimizing the processing parameters. A “nice” multilook impulse response is achieved by averaging distorted FIRF, at the cost of a significant reduction in the processed Doppler bandwidth. (hence, in ENL).

Thereafter, a different approach was introduced, where ScanSAR inverse MMSE kernels were designed, resulting in minimal distortions within a few resolution cells from the footprint boundary. For multilook detected images, the gain over optimized conventional techniques is essentially due to the larger Doppler bandwidth exploited (85% of the processed Doppler bandwidth in GMM mode) and to a ~3 dB improvement in PSLR (or a better resolution). For interferometric applications, an improvement of interferogram SNR of up to 7 dB (ASAR WSM case) comes from getting stationary FIRF. The computational overhead is not heavy, since MMSE reduces to a short matrix multiplication, that could be combined as a preprocessor to any existing focusing scheme.

APPENDIX A
SCANSAR INVERSION

The focusing of SAR data has traditionally been tackled with the matched filter theory. However, there are some examples in the literature (e.g., [14]) that frame this problem as an inversion problem.

Let us briefly investigate the feasibility of inverting the ScanSAR acquisition (7)

\[ u(t) = \int_{-\infty}^{+\infty} h_s(t, \xi) x(\xi) \, d\xi \]  

(13)
e.g., reconstructing the unknown reflectivity \( x(\xi) \) given the burst signal \( b(t) \) and the ScanSAR IRF \( h_s(t, \xi) \) in (1). If the inverse operator \( h_{is}(t, \xi) \) exists, it satisfies the following condition:

\[ \int_{-T_D/2}^{+T_D/2} h_s(t, \xi) h_{is}(t - t, \xi) \, dt = \delta(t - \xi). \]  

(14)

Equation (14) is satisfied if \( T_D \to \infty \), by the following solution (stationary phase method is adequate):

\[ h_{is}(t, \xi) = a_{1/0}^{-1}(t - \xi) \exp(-j\pi f_R(t - \xi)^2). \]  

(15)

If \( T < \infty \), no \( h_{is} \) can be found, as the direct operator becomes low pass

\[ \int_{-T_D/2}^{+T_D/2} a_{1/0}(t - \xi) \exp(j\pi f_R(t - \xi)^2) \cdot \exp(-j2\pi ft) \, dt \to 0 \text{ as } f \to \infty. \]  

(16)
x(t) cannot be exactly recovered and it is necessary to resort to approximations. The same need arises if noise is added to data.

One classical way is to look for the MMSE solution of (13)

\[ \min_{x(\xi)} \left\| \int_{-\infty}^{+\infty} h_s(t, \xi) x(\xi) \, d\xi - b(t) \right\|^2. \]  

(17)

Other regularization procedures can be devised to modify (17) [13].

Conventional SAR differs from ScanSAR only because the direct operator is invariant (i.e., \( h_s(t, \xi) = h_s(t - \xi) \)). (13) is a convolution, and the inverse operator is easily found in the frequency domain. Here the AAP restricts the temporal width and the bandwidth of the operator. Therefore, the inversion has to be restricted only to the received signal bandwidth. The recovered reflectivity is, therefore, an MSE approximation to the true reflectivity \( x(t) \).
Fig. 11. (a) ScanSAR MMSE inversion has been decomposed in the matrix product of two operators: the SAR matched reference and a distortion remover. This second operator contains the matrix inverse of SAR auto-covariance. Only when $M$ becomes large does it defaults to a LTI filter. (b) Up: auto-covariance of ScanSAR (or SAR) data, sampled at a rate $1/PRF$, and down: the corresponding inverse operator. It extends beyond the burst data and therefore, cannot be applied.

**Appendix B**

**Connection Between MMSE and Conventional Focusing**

Let us simplify the Wiener inversion (11) to retrieve the best resolution and a flat amplitude over the largest bandwidth e.g., $R_x = I$

$$\hat{X} = \mathbf{H}_s^*(\mathbf{H}_b \mathbf{H}_s^* + R_n)^{-1} \mathbf{B}. \quad (18)$$

Let us define the distortion remover operator

$$R_1 = R_B^{-1} = (\mathbf{H}_b \mathbf{H}_s^* + R_n)^{-1}. \quad (19)$$

MMSE inversion (18) becomes

$$\hat{X} = \mathbf{H}_b^* R_1 \mathbf{B}. \quad (20)$$

It is schematically represented in Fig. 11(a). The multiplication by $\mathbf{H}_b^*$ is the correlation with the matched SAR reference. Usually $\mathbf{H}_s^*$ is long since output data $\hat{X}$ is larger than the burst duration. However, the leftmost matrix product in (20), the most time consuming, can be implemented by any efficient FFT-based techniques [7], [8]. The short matrix multiplication by $R_1$, shown in Fig. 11(a) is exactly the linear, generally time variant operator that converts the matched-SAR filter $\mathbf{H}_b^*$ in the MMSE ScanSAR inverse.

A. AAP Compensation by a Time-Varying Filtering

For a better understanding of MMSE in ScanSAR focusing, we need to evaluate the autocorrelation matrix $R_h = \mathbf{H}_b \mathbf{H}_b^*$ involved in (18). Let us evaluate each element of matrix $R_h$ by using the ScanSAR acquisition model (1) and (6)

$$r_h(t_m, t_n) = \sum_{t} h_a(t_m, \xi) h_a^*(t_n, \xi) = \sum_{t} a_\beta(t_m - \xi) \text{rect}\left(\frac{t_m}{T_D}\right) \cdot \exp(jf_R(t_m - \xi)^2) a_\beta^*(t_n - \xi) \text{rect}\left(\frac{t_n}{T_D}\right) \quad (21)$$

Let us find a suitable approximation for the summation in (21). We assume here that the summation vanishes for $|t_m - t_n| > 4/PRF$, e.g., that its extent is as short as four full resolution samples. We will verify this assumption a posteriori. In that case, the rectangular burst windows in (21) are ineffective and they can be dropped. Moreover, the AAP $a_\beta(t)$ does not change appreciably. We can introduce the change of variable $\Delta_k = t_n - t_m$ and thus approximate (21) as follows:

$$r_h(t_m, \Delta_n) \simeq \exp(-j2\pi f_R t_m \Delta_k) \cdot \sum_{t} |a_\beta(t_m - \xi)|^2 \exp(j2\pi f_R \xi \Delta_k) \quad (22)$$

where we acknowledge in the summation the FT of the time-shifted and sampled, squared AAP. For a conventional rectangular SAR antenna, the AAP can be approximated as follows:

$$a_\beta(t) = \text{sinc}^2\left(\frac{t f_R L}{2\nu}\right) = \text{sinc}^2\left(\frac{t f_R}{PRF/\rho}\right) \quad (23)$$

where we have assumed that the SAR data sampling step $\nu/PRF = L/(2\rho)$ is slightly smaller than resolution, where $\rho \simeq 1.25$ (that holds for ERS and ENVISAT). If we define the triangular shape as $t(t) = \text{rect}(2t) \times \text{rect}(2t)$, we get

$$r_h(\Delta_k) \propto \text{tr}(\Delta_k \rho PRF/2) \times \text{tr}((\Delta_k \rho PRF/2) \quad (24)$$
The autocorrelation of the triangular waveform is a Gaussian-like shape (a cubic B-Spline) with duration limited to $4p_{\text{PRF}}$. Sampling $\tau_k$ at time interval $\Delta_k = k/\text{PRF}$ gives the three (full resolution) sample sequences

$$A_2(k) \propto \alpha \delta(k+1) + \delta(k) + \alpha \delta(k-1)$$

where

$$\alpha = \frac{1}{4} + \frac{3}{4} \frac{1-p}{p} + \frac{3}{4} \left(\frac{1-p}{p}\right)^2 - \frac{3}{4} \left(\frac{1-p}{p}\right)^3 = -0.43.$$  \hspace{1cm} (26)

This verifies the initial assumption made in deriving (22) from (21). The obvious conclusion is that $\tau_k(m,n)$ keeps the value of the autocorrelation of ScanSAR raw data (in azimuth), which is, however, equal to that of SAR data. The value of $\alpha$ (or its related parameter $\rho$) involved in computing $R_B$ is usually known within high accuracy, given the AAP and the sampling. Only the Doppler centroid is required to compute $R_B$ in the real case (here assumed to be zero for simplicity). However, this one can be tracked accurately [3].

With reference to (19), (20), and Fig. 11(a), let us define

$$\mathbf{B}_n = R_B^{-1} \mathbf{B} = R_T \mathbf{B}$$ \hspace{1cm} (27)

the burst deconvolved by the squared AAP. If we assume $R_n = \mathbf{0}$ then $R_B = R_T$ is a Toeplitz matrix with only three nonzero diagonals. They contain the sequence in (25) and (26). If we ignore the border effects (whose extent is only one pixel in the very first and last rows, $R_B$) then plays the role of a LTI filtering with a filter that has a three sample impulse response, as shown in Fig. 11(b) (upper plot). The distortion compensation operator $R_B$ is then the inverse reference. Theoretically, the short sequence (25) would be inverted (or deconvolved) by an indefinite reference one, yet, as Fig. 11(b) shows, that inverse reference can be practically truncated to 40 full resolution samples. This number results from assuming $\rho = 1.25$ (this holds for ASAR) in (23), (25), (26), and roughly corresponds to the length of the burst if we impose $TB_{BP} = 1$, e.g., $N = \sqrt{TF_{PRF}} = 40$, according to the values in Table II.

1) Large $TB_{BP_{az}}$: In this assumption, the burst is rather long compared to the inverse reference in Fig. 11(b). Hence, $R_T \mathbf{B}$ in (27) defaults to an LTI filtering.

$R_B$ is Toeplitz. We can well approximate it as circulant (the sole difference being one element in the first and last rows). In that case, it is orthogonally diagonalized by the DFT matrix $\mathbf{T}_N$

$$R_B \simeq \mathbf{T}_N^* \mathbf{A} \mathbf{T}_N$$ \hspace{1cm} (28)

where $\mathbf{A}$ is the eigenvalues (diagonal) matrix, whose elements $\lambda_k$ can be computed as $\text{DFT}^{-1}$ of the sequence $A_2(k)$ in (25), giving the result in (23). The addition of the diagonal, white noise matrix $R_n$ in (19) results in adding noise power to the eigenvalues $\lambda_k$. Henceforth

$$\lambda_k \simeq \text{sin}^4 \left(\frac{k \rho}{M}\right) + \frac{P_n}{M}, \quad -M/2 \leq k \leq M/2.$$ \hspace{1cm} (29)

The inverse of $\mathbf{T}_N^* \mathbf{A} \mathbf{T}_N$ can be easily obtained as the DFT matrix is unitary

$$R_T \simeq \mathbf{T}_N^* \mathbf{A}^{-1} \mathbf{T}_N.$$ \hspace{1cm} (30)

This suggests another way to apply the distortion remover operator, the rightmost matrix multiplication in (20). It is sufficient to take the DFT of the burst, multiply by the sequence $\lambda_k^{-1}$ derived in (29), and back-transform. This operation can be efficiently implemented by means of FFT provided that the burst length has a suitable prime number factorization.

If (30) holds, the FT of the inverse operator is, according to (24), (19), the inverse of the SAR power spectrum $R_s/f = \left(\alpha^2_j(f)P_s + P_n/M\right)^{-1}$. This provides the correct AAP deconvolution, which combines with the matched reference $\mathbf{H}_s^*$ in (20) to give an overall transfer function amplitude (including acquisition and focusing)

$$\frac{\alpha^2_j(f)P_s}{\alpha^2_j(f)P_s + P_n/M}$$ \hspace{1cm} (31)

which converges, in the noiseless case ($P_n = 0$) to the solution provided by the SAR inverse; the PCF reference. Note that the solution does not change if the noise level $P_n/M$ falls below the antenna gain at the footprint edges ($\pm 15$ dB). In this case, Wiener inversion defaults to the LS inversion of the ScanSAR acquisition: $\mathbf{B} = \mathbf{H}_s \mathbf{X}$.

It is worth noting that the same inverse reference implied in (30) is the one used by conventional focusing: $\approx 40$ samples inverse reference in the example of Fig. 11(b). Clearly, that inversion is more effective when the data extent (the burst duration) is $>>$, than when the inverse reference and no one cares of border effect.

2) Small $TB_{BP_{az}}$: Let us then move to the case $TB_{BP_{az}} \sim 1$. The direct problem $\mathbf{B} = R_B \mathbf{B}_k$ is always inverted by matrix algebra in (27) whatever the burst length, e.g., independently on the size of $R_B$ since all its eigenvalues are positive, according to (25), (26), (29). The inverse reference $R_T$, which can be derived by the approximation in (30), is Toeplitz$^4$ circulant [see Fig. 11(b)]. Hence, the antenna pattern whitening is now performed by a linear, time-varying operator.

On the other hand, in the conventional focusing, the long inverse reference is shifted and truncated in the convolution with the short burst. This shift + truncation strongly distort the focusing operator, in a way that changes at each pixel: this is the fundamental cause of the nonstationary distorted FIRF output.

One can wonder if MMSE inversion is still capable to provide, in this case (small $TB_{BP_{az}}$) good FIRF shape, in terms of 1) peak height, 2) peak location, 3) desired sidelobes attenuation, and finally, 4) FIRF symmetry.

The first two properties can be shown by ignoring the noise matrix $R_n$ (that is indeed small), then MMSE becomes the LS pseudo-inversion. We get from (18)

$$\hat{\mathbf{X}} = \mathbf{H}_s^* (\mathbf{H}_s \mathbf{H}_s^*)^{-1} \mathbf{B} = \mathbf{H}_s^* (\mathbf{H}_s \mathbf{H}_s^*)^{-1} \mathbf{H}_X \hspace{1cm} (32)$$

$$\mathbf{H}_s \hat{\mathbf{X}} = \mathbf{H}_s \mathbf{H}_s^* (\mathbf{H}_s \mathbf{H}_s^*)^{-1} \mathbf{H}_X \Rightarrow \mathbf{H}_s \hat{\mathbf{X}} = \mathbf{H}_X.$$ \hspace{1cm} (33)

$^4$A more rigorous derivation from (25), (26) shows that the actual inverse is not even Toeplitz.

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Then $\hat{X}_e = X$ in the norm of $H_e$. The solution always exists since the matrix $H_e H_e^*$ is nonsingular, as it comes out from (25), (26) for $\rho > 1$. When $H_e$ is square, e.g., if we compute as many output samples as the number of echoes in the burst, then the pseudo-inversion becomes exact: $\hat{X}_e = X$.

In the case when the output is required at a finer sampling then the actual resolution, we get an undetermined system, and the LS pseudo-inversion performs the necessary interpolation. The impulse response FIRF is always Hermitian symmetric due to the Hermitian symmetry of the projection matrix $H_e^* (H_e H_e^*)^{-1} H_e$ in (32) [23]. However, one has to account for aliasing, e.g., the symmetry holds if the output is periodically folded. The symmetry of the solution ensures the correct peak location. Hence, properties (2) and (4), and this is quite evident in the plots in Fig. 6(b). For what concern the peak height (1), it appears from Fig. 6(a) that there is a small bias getting from the center of the footprint to its end. There is a simple explanation for this fact, that is due to aliasing of the antenna pattern. The pseudo-inversion in the discrete time domain does effectively compensate the folded AAP, in place of the true one. Hence, when approaching the footprint end, the true antenna gain is slightly overestimated, therefore the inversion tends to undercompensate and the restituted amplitude drops. This biasing is not a problem, since it is indeed small ($<1\text{ dB}$) and, if desired, it can be estimated and compensated for. It would disappear in the multilook average regardless.

**ACKNOWLEDGMENT**

The authors would like to thank the European Space Agency (ESA-ESTEC), Noordwijk, The Netherlands, and the Italian Space Agency (ASI), Rome, Italy. They would also like to thank Prof. F. Rocca for his helpful discussions, and also Prof. C. Cafforo, who built up the verification mode ScanSAR processor. He was the first to assess the FIRF artifacts in low TBP focusing, and he provided an extensive and accurate revision of the whole paper.

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